Numbers, Mathematics, and Rhythm

A Numerical Approach to Composition

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January 2021
Funded by Arts Council England – ACPG-00304930

Outline

The purpose of this paper is to explore the possibilities of adopting a numbers-based approach to composition. Bob Becker's **Rudimental Arithmetic** (RA) proposes numerous mathematical devices which can be employed during the composition process. Furthermore, number-based aleatorics can play an interesting role in the development of phrase, structure, and rhythmic cycle. In this paper I will explore these concepts and how they might be employed in new compositions, and discuss the relative musical merits of such a course.

I. Introduction

Elsewhere, I have discussed at great length the various conceptual approaches available to the composer when enacting their craft.[1] In that paper, the broader concepts of aleatoricism and its bearing on composition are addressed. That paper also includes detailed case studies of some of my own work in which numbers-based aleatorics are applied. Here, we shall endeavour to explore the more practical side of the idea of numbers and mathematical relationships, and how these can be applied to the generation of musical content. This paper, then, is the more practically-minded sister of that one; a sister which relies on her sibling's knack for ideas while really rather preferring getting her own hands dirty in the doing.

Broadly, then, this approach uses numbers, mathematics, and numerical relationships to define, create, link, or otherwise manipulate musical phrases. We could reasonably be forgiven for thinking that this approach seems somewhat at odds with the more traditional ways in which composers write. Pratt, for instance, is known for his use of strong, recognisable rudimental phrases which are introduced, varied, expanded upon, and concluded in very musical and pleasing ways. Delécluse, on the other hand references the classic orchestral repertoire whilst leaving ample room for the performer to apply their own interpretation of the notes on the page.

These approaches, it is clear, are generally not based on any mathematical principles, rhythmic cycles, or other such devices, yet are rightly considered paragons of snare drum composition in their own rights. The numerical approach herein described instead presents a concept in which the relationships between the phrases, their numerical creation, and the fascinating mathematical interlinking that results is at the heart of the music. There is something inherently pleasing about things that neatly align, resolve, or conclude over time following a set of rules or principles. There is joy to be found in the knowledge that a particular phrase is born from the result of the interplay between two other phrases found previously in the piece.

[1] (Curtis, Snare Drum Composition – Traditions, Concepts, and Approaches, 2021)

We should address here that we are firmly entering the realm of what is today called *art music*.[2] This seems to be a relatively modern term used to describe music in which the construction, structure, theoretical considerations, and various other aesthetic elements of the music are held and revered at least as highly as how it all sounds. While I must confess little interest in the etymology of the term, and even less in the implicit elitism that seems to come along with it, I can allow that such elements are appealing to me.[3] As with mathematicians who feel a sudden pang of comradery with the universe upon the discovery of a fundamental formula or principle, there does seem (to me at least) to be something a little transcendental about playing music containing some degree of underlying mathematical principle. Playing a figures comprised from a descending series of prime-number partitions, for example, feels just a little like playing on the skin of the universe itself (if you'll forgive a rather artistic sentiment).

As a note of terminological clarification, I am using the word *aleatoric* in a way perhaps not originally in keeping with its definition. I shall neglect defining the word and presenting its history here, but I am nevertheless coining it and bending it to my own will. For this paper, as in any in which I discuss this concept, *aleatoricism, aleatoric,* and *aleatorics* describe some compositional process based on systems and devices outside the direct intention of the composer. While this may include chance elements, as expressed in the traditional definition of the term, this can also include formulaic or algorithmic devices, or, as is the case here, mathematical processes. The crucial element is that the results are derived from the processes, systems, or devices, rather than from the particular intention of the composer. If a composer determines to follow a mathematical principles to derive their structure, ignorant of exactly what structure will be produced, I consider that to be an aleatoric approach.

To wit, we shall park for now the sentimental or conceptual considerations and turn our full attention to the practical nature of this craft.

II. Developing Phrases from Numbers

We begin with the number 24. For our current purposes, this choice of number is entirely arbitrary. A numerically inclined composer may choose a number for a host of reasons, from numbers at the heart of an overarching conceptual framework, to a loved one's birthday, or simply a favourite number. That choice shall be reflected in their underlying compositional principles which (we must remain strict!) do not concern us here.

24 possess a multitude of mathematical characteristics which are not immediately obvious. To list a few, 24 is:

- Even
- The factorial of 4 (there are 24 permutations of 4 distinct elements)
- Composite (non-prime)
- A highly composite number, having more divisors than any smaller number
- The sum of prime twins 11 and 13
- The total number of major and minor keys in Western tonal music[4]

Some of these are known to almost everybody. That 24 is even is of little remark; that is until we start to see what import this has on how it can be manipulated and summed. Any

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[2] ...or sometimes *cultivated*, *canonic*, or *aesthetic* music.

[3] I have spoken in this paper's sister work about the philosophical merits of such music. I shall not belabour the points here.

[4] This has little immediate bearing on our current study, but was instrumental in Chopin's 24 *Preludes*.

even number, for instance, can be the product of two odd numbers, such as the twin primes of 11 and 13.

Let us consider this in more detail: *24 is the sum of prime twins 11 and 13*. The numbers 11 and 13 are twinned primes,[5] and adding them together equals 24. We shall pair this with another of the characteristics listed: *24 is the factorial of 4*. Using these two characteristics, we can begin to create some interesting and mathematically linked phrases.

Considering that 24 is the factorial of 4, we can begin by finding four distinct elements. We can derive these from the prime numbers 11 and 13. As we are trying to derive four distinct elements, we can assign two each to both 11 and 13.

Beginning with 11, it can be partitioned in the following ways:

10+1	7+4
9+2	6+4
8+3	

13 can be partitioned in the following ways:

12+1	9+4
11+2	8+5
10+3	7+6

Looking at these partitions, we can choose one from each list. This will once again be a philosophical choice for the composer, but for us it shall be entirely arbitrary. For 11, we shall choice 8+3, and for 13 we shall choose 9+4.
Thus, we have arrived at the numbers 8, 3, 9, and 4 via a simple process. Firstly, 24 was divided into its twinned primes 11 and 13 (24 being the sum of these two numbers). 11 and 13 were both then divided again into their respective partitions of 8 and 3 for 11, and 9 and 4 for 13, creating the four elements 8, 3, 9, and 4. Recall that we wanted four elements to begin with due to the relationship between the number 4 and its factorial, 24.

With this groundwork duly undertaken, we can begin creating a musical phrase. More philosophical questions arise as to the context in which these elements should be framed: should we create polymetric measures as we progress through each figure? Should these elements correspond to subdivisions or two-part polyrhythms? The options are well-nigh limitless (notwithstanding the ever-present notion that music should, to greater or lesser extent, remain listenable and somewhat comprehensible lest we nose dive down the postmodernist rabbit hole).

As it so happens, when taken as 16th notes, 24 notes equate to one measure of 12/8:

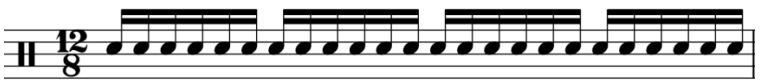


Fig. 1.1

Taken our four chosen elements, we can mark out the figures using accents:

[5] Being prime numbers separated by 2.

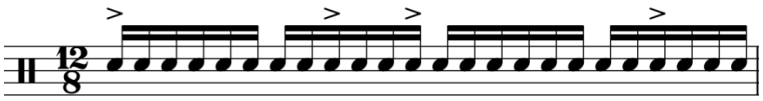


Fig. 1.2

Each grouping in Fig. 1.2 corresponds to the sequential elements 8, 3, 9, and 4. If so inclined, marcato accents can be used to mark the twin primes 11 and 13 to create secondary groupings:

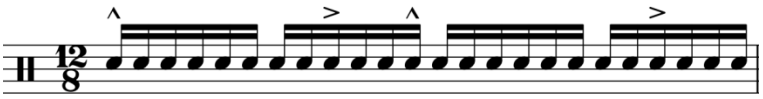


Fig. 1.3

Using simple rudimental sticking, a phrase is formed, corresponding to the numerical phrases we have developed:

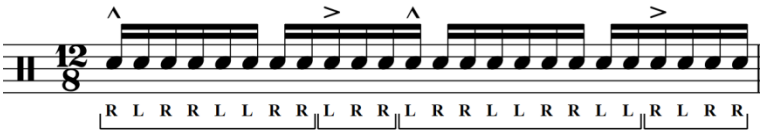


Fig. 1.4

We have at last arrived at some tangible musical content, entirely derived from numerical aleatorics.

We can see from the resulting figure, above, that the phrase created is alternating. This means that, if we were to repeat the same phrase in the next bar, we would begin with the left hand and reverse all of the subsequent sticking.

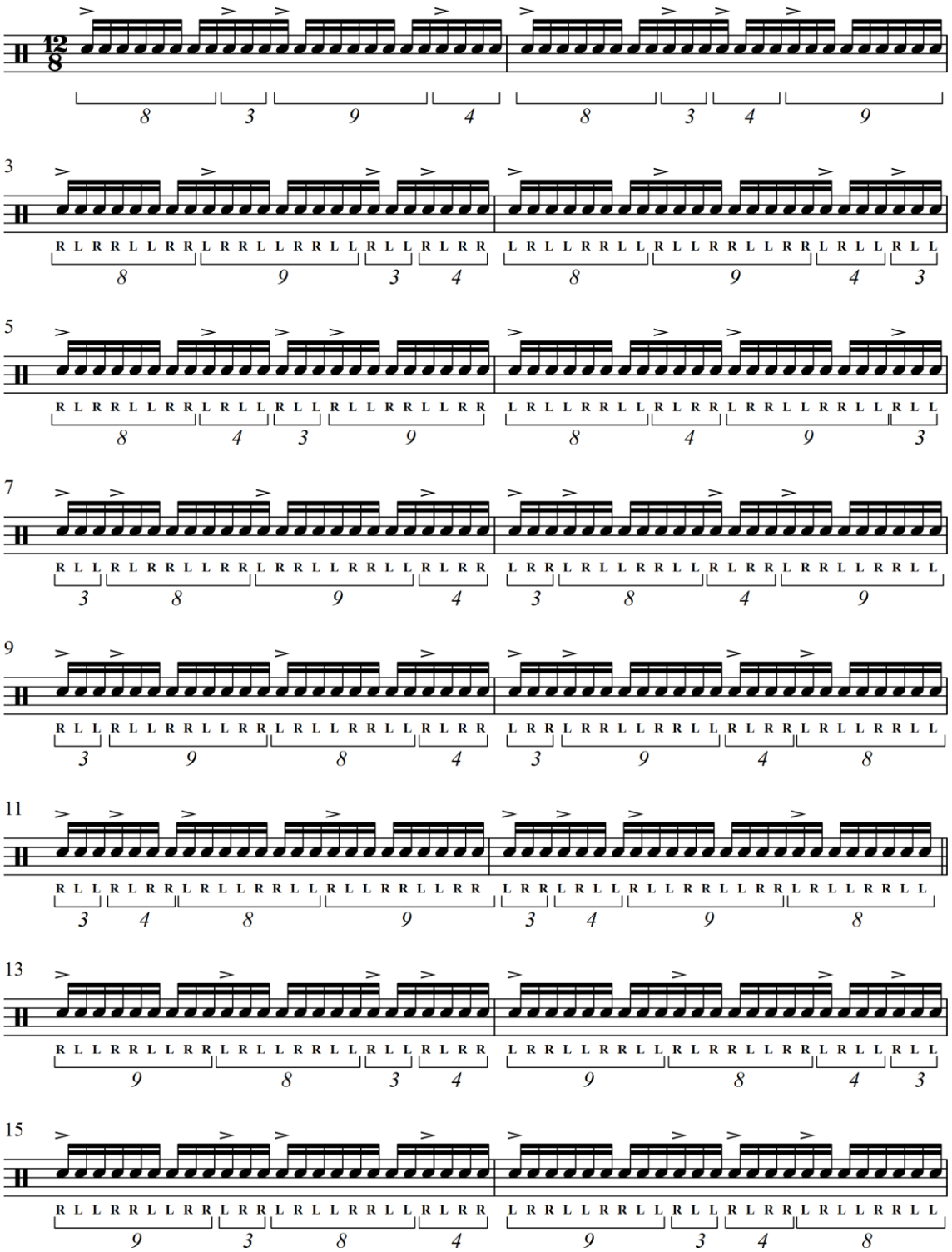
At this point, the possibilities increase exponentially. We know from our previous explorations that 24 is the factorial of 4, and we have here a phrase with 4 constituents. We could, for example, formulate 23 further variations of this bar utilising these permutations. Currently in Fig. 1.4, our phrase runs 8, 3, 9, 4, which is just one of these 24 possibilities. This itself is somewhat satisfying. The resulting piece is 24 bars of 24 16^{th} notes; each bar comprised of one the 24 permutations of the four elements numerical derived from 24, itself the factorial of 4.

Etude #2

24 Permutations
Skeleton Structure

Jonathan Curtis

♩. = 96



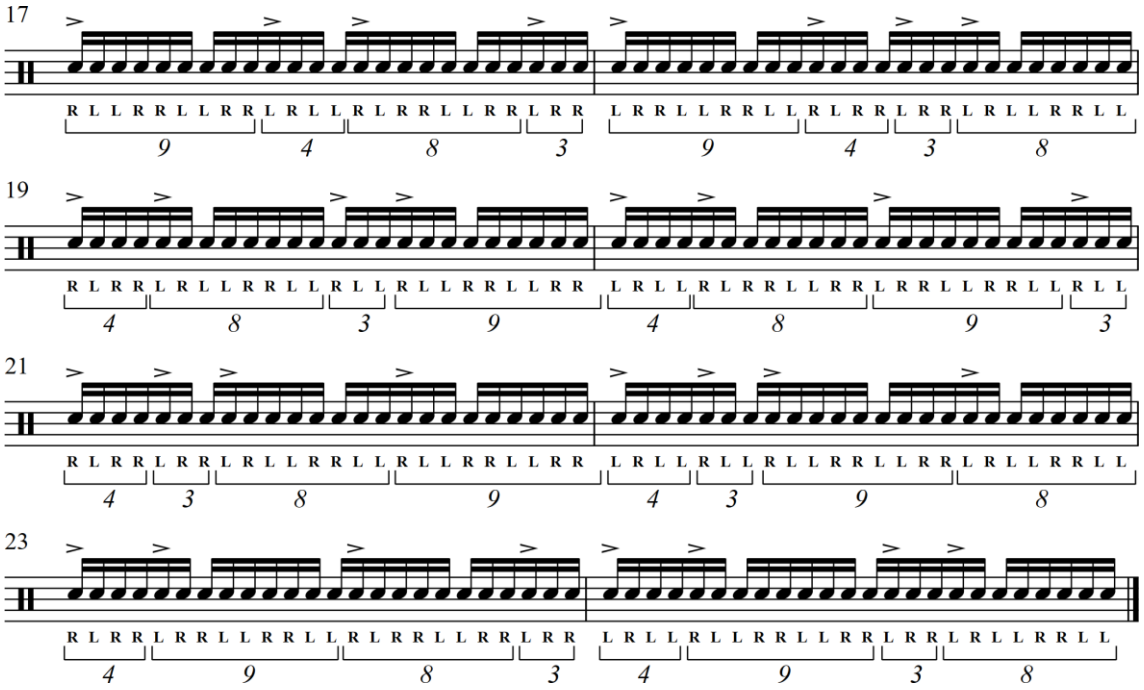


Fig. 1.5

Here, then, is a simple etude derived from this process. Each bar represents a unique permutation of the four elements which we derived from the number 24. The stickings have been applied using diddle figures and measured rolls, and reversed where necessary to allow for fluid playing. The tempo is set at **96 bpm**, being a multiple of 24, though this is perhaps labouring the point somewhat.

From a compositional perspective, this stage marks the end of the beginning. A structure has been created according to aleatoric principles; the number of measures, the time signature, the rhythmic groupings, all bear some relation to the number 24 which began it all.

As an etude, this serves well. It presents a compositional concept which provides an ample technical challenge. It has a musical form and structure, but lacks higher levels of musicality, and it is to this that we now must turn.

III. Phrase Development

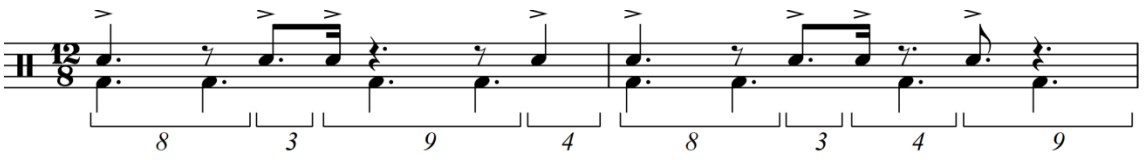


Fig. 1.6

With the skeleton structure now laid out as in Fig. 1.5, we can begin a deconstruction of the figures, reducing them to a series of accents. Fig. 1.6, above, shows the first two bars of our new etude as an accent pattern. The 16th notes between each accent have been removed.

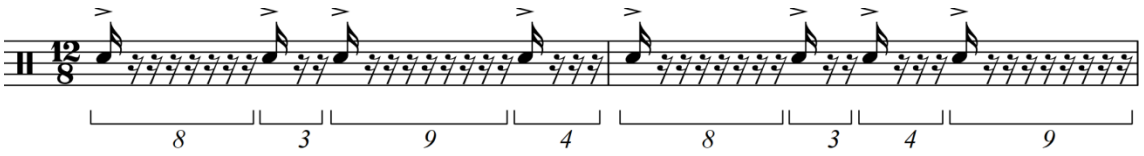


Fig. 1.7

This is perhaps more easily visualised using 16th notes throughout the phrase, though this is of course impractical for most purposes. We can see above the groupings as they would appear in this format. More specifically, we can see the relative *value* of each figure very clearly throughout the phrase, assigned as a number referencing the number of 16th notes comprising each figure.

With this in place, we can begin various manipulations of the musical content based on these values. Becker's work on phrase values is of enormous interest to us here.[6] Becker codified the three rudimental families of drags, diddles, and measured rolls in a way which assigns them all values based on how many beats they occupy. A 16th note paradiddle, for example, is assigned a value of 1 for binary time (occupying one quarter note), and $^{2}/_{3}$ in ternary time, occupying two thirds of a dotted quarter note. This work allowed him to present a concept of phrase construction based on substituting figures of equivalent values within a frame work. Taking this work as a basis, we can explore the concept of values and substitution with regard our current figures.

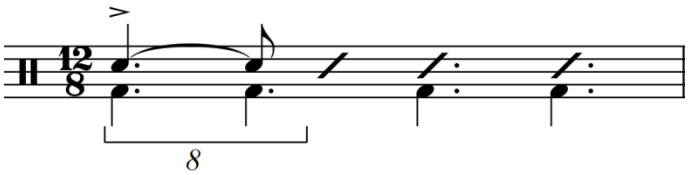


Fig. 1.8

The 8 we see above is shown here relative to the underlying pulse. Under Becker's taxonomy, a phrase of this length in this context would possess a value of $1^{1}/_{3}$. A dotted quarter note is conceptualised as containing three 8th notes, each comprising one third of the beat. As can be seen above, eight 16th notes occupy the entire first beat, and the first 8th note of the second, thus the value given. Staying with Becker for now, we can insert into this space various rudimental forms possessing the same value.

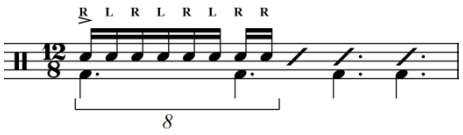


Fig. 1.9 – Triple Paradiddle

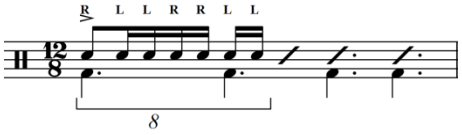


Fig. 2.1 – 7-stroke roll

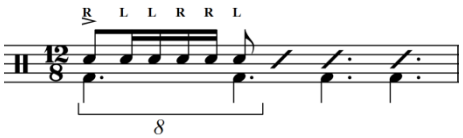


Fig. 2.0 – 6-stroke roll

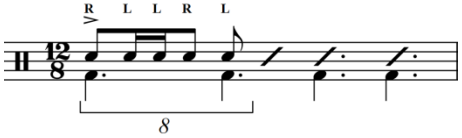


Fig. 2.2 – Lesson 25 Drag

[6] (Becker, 2008)

All of the above examples substitute figures with the same value of 1⅓ relative to the 12/8 meter. While by no means exhaustive, these four examples show how the use of different rudimental vocabulary can alter the overall feel of the figure.

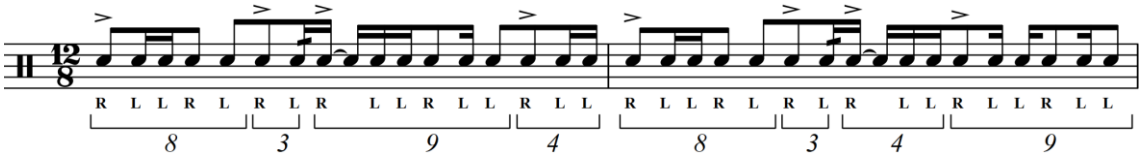


Fig. 2.3

In the above phrase, the first two bars of our new etude have been modified utilising this concept of value substitution. The figures of the first measure have been replaced with drag figures of the corresponding value, and these have been continued into the second measure and rearranged according to the previously determined permutation structure.

Already it is clear how powerful this approach can be. The phrase shown in Fig. 2.3 is rhythmically interesting, challenging yet idiomatic to play, and falls entirely within the aleatoric principles herein presented.

Expanding upon this theme, we come to another of Becker's ideas: rhythmic density. Figures of any length can be moulded into varying densities based on the note values used within them. A figure is said to be at its lowest density when all of the strokes are of equal time value against the beat.[7] Typically, lowest density rolls are equal to their number, such that a five-stroke roll would contain five 16th notes. This can be played as a tuplet against the beat, such that a five-stroke roll would comprise one 16th note quintuplet against a quarter-note beat.

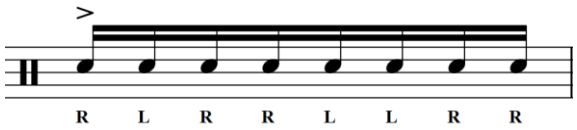


Fig. 2.4

Above, we see an 8-stroke roll in its lowest density.[8] The accent and subsequent strokes are all 16th notes.

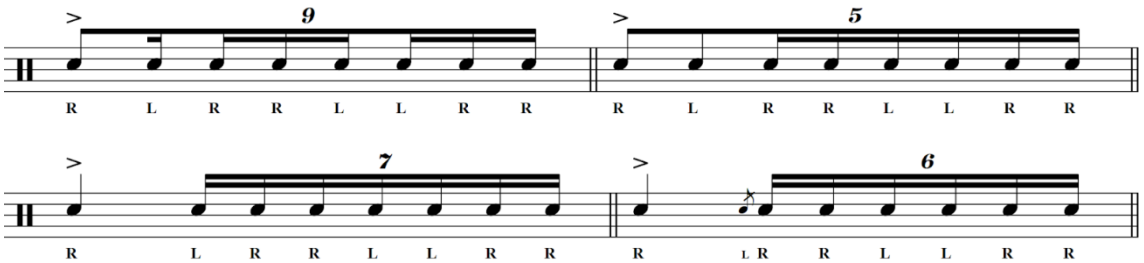


Fig. 2.5 [9]

Fig. 2.5 shows the same rudiment presented across varying density forms. These are delightful manipulations of the original figure, and create a wealth of interesting

[7] (Becker, 2008), p.16
[8] We could arguably present a version as 32nd notes against a single quarter note, but this is effectively the same thing as far as the rhythmic density of the figure is concerned.
[9] (Becker, 2008), p.19

opportunities for applying an otherwise rather simple rudiment. All of the above figures retain the original value (a half note), yet the internal rhythms have been altered.

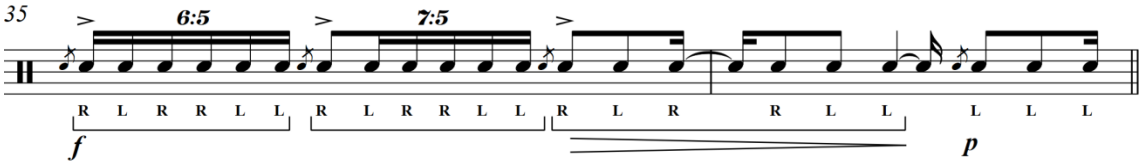


Fig. 2.6 [10]

In the figure above we seen an example of the concept applied in my own work. The meter for this piece is 15/16, referencing three groups of five 16th notes per measure. In this example, a paradiddlediddle is manipulated through various densities, polyrhythmically across the underlying pulse and over the barline.

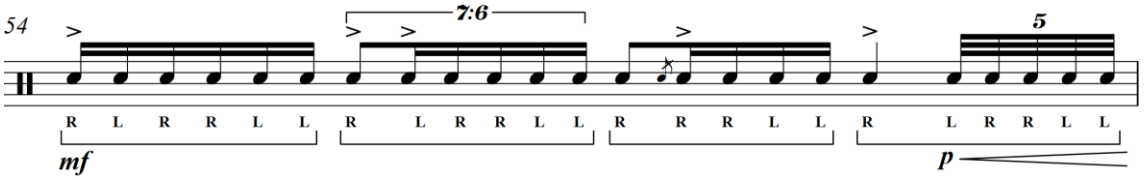


Fig. 2.7 [11]

Above is another example of this concept in my own work. This time the meter is 12/8, with an underlying pulse of six 16th notes per beat. A paradiddlediddle is once again manipulated throughout the measure using different densities. We can see the same sticking played four times sequentially (including the left hand grace note in the third figure's flam), with a different internal density each time.

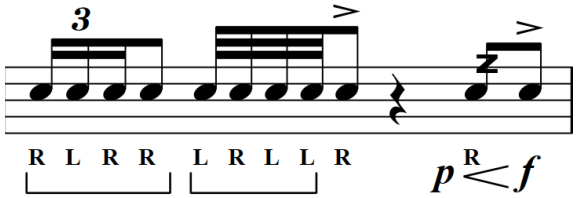


Fig. 2.8 [12]

Fig. 2.8 shows another example of this concept from my piece *Dekatría*. Here, a paradiddle is phrased with two different densities in 4/4 time to lead into the accent on the *and* of the second beat.

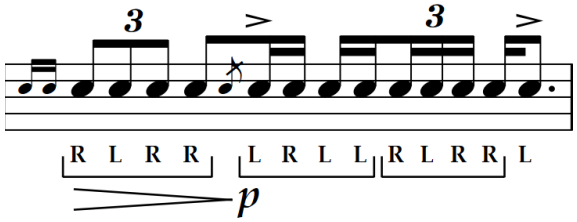


Fig. 2.9 [13]

[10] (Curtis, Numinous Measure, 2021), bars 35–36, © Artificer Productions
[11] (Curtis, Rata Path, 2021), bars 54–55, © Artificer Productions
[12] (Curtis, Dekatría, 2021), bar 29, © Artificer Productions

Finally in Fig. 2.9, above, we see another example from the same piece in which three paradiddles are played sequentially through different densities, again in 4/4 time. The resultant phrases are not only interesting to listen to, but coherent to play. Their understanding allows for high degrees of manipulation of figures within a larger rhythmic structure.

Returning to our current study, we have in our newly derived phrase accents as starting points:

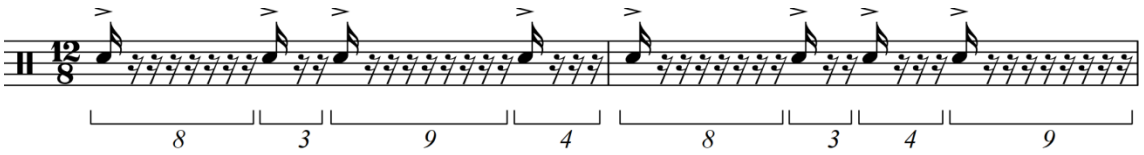


Fig 1.7

Though it is important to consider the value of the groupings, we must be careful in our application of this rhythmic concept.

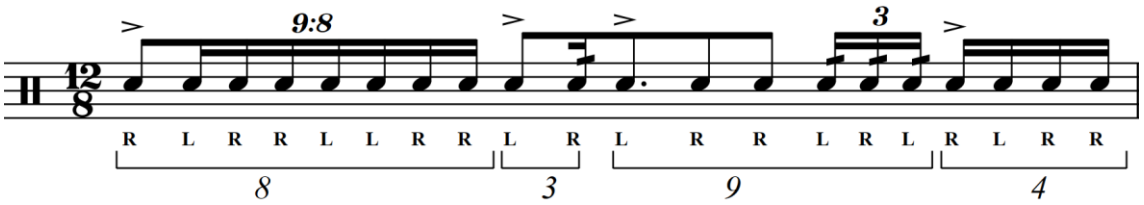


Fig. 3.0

Above, the densities of each figure have been changed in way which preserves the location of each accent, but the result is largely incomprehensible. Aside from being extremely difficult to read within the given meter, the underlying pulse is lost and we have instead a rhythmic soup which verges on ametric.

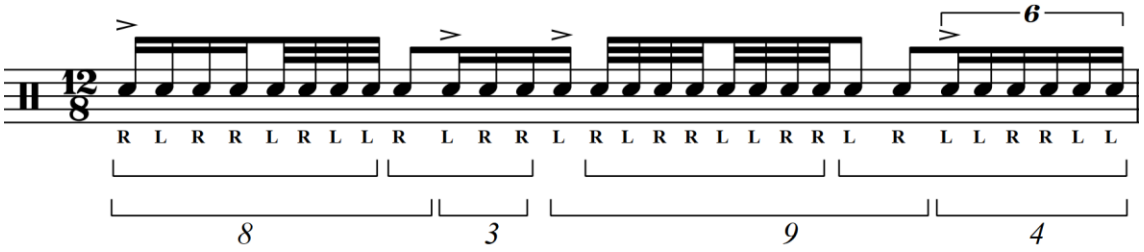


Fig. 3.1

Here instead is a slightly more elegant example, in which the concepts of densities and phrase values have been employed whilst retaining legibility and a sense of pulse. This example still allows for interesting rhythmic elements, such as the polyrhythm in the fourth beat, yet strikes a more pleasing balance between concept and practice.[14]

Becker suggests that a numerically inclined composer can create interesting phrases if they possess working knowledge of such procedures:

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[13] (Curtis, Dekatría, 2021), bar 42, © Artificer Productions

[14] A dichotomy I discuss at greater length in (Curtis, Snare Drum Composition – Traditions, Concepts, and Approaches, 2021)

> *"Familiarity with the bases for determining pattern values, permutations of elements and groups, and configurations of polyrhythms is necessary in order to obtain the greatest benefit... the concepts being discussed are fundamental to understanding and appreciating both the intricacies of two-handed playing and the immense number of possibilities inherent in simple patterns... especially when structured by a background awareness of a specific meter and, ultimately, larger forms of organisation as well."* [15]

The finished etude displays a variety of these conceptual themes running through the four sections.[16]

In this paper's twin, I discuss the balance between following systemic processes for their own sake and coherent musicality, but the main point bears repeating here. I believe it is of the utmost importance that composers should not be slaves to the concept at the cost of musical coherence. There comes a point beyond which the musical content simply does not sound good. That point is a subjective one and will be different for each composer and every listener, and various composers have followed their philosophical convictions to their logical conclusions. The relative musical merit of such a path is beyond the scope of this paper, and is touched upon elsewhere, but I retain a firm conviction that ultimately I want for my compositions to be enjoyable to listen to, despite (or in spite of!) any conceptual processes I may have utilised in their creation.

The musical examples in Figs. 2.3 and 3.1 were crafted solely by following fundamental principles: structure derived from aleatoric means, and figures derived from underlying metric values. It falls to the composer to embellish, modify, and in all ways manipulate these creations such that they are in some way musically coherent and sympathetic to both performer and listener. Stickings may be modified for the sake of ergonomic fluidity; dynamic peaks and troughs added; internal rhythms can become themes to be reprised subsequently; flams, drags, rolls, and other rudimental embellishments can be added to flesh out the figures; and we have said nothing, expression, broader thematic development, or structural narrative. To me, these aspects are applied at the very next stage from the one arrived at in this paper. I often describe my creative act as akin to a sculptor; I begin by moulding the clay into a rough shape, before making many passes fine tuning the details.

The above process has forged the overall shape and applied a first pass; the musical touch ups from hereon in are potentially infinite and absolutely at the whim of the composer, and as such would make for a tedious and rambling analysis if attempted here.

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[15] (Becker, 2008) pp. 54–55

[16] (Curtis, Etude #2 – 24 Permutations, 2021)